source of trouble, since in his later editions it is simply omitted, though as before his proof depends upon it.

His treatment of tangent circles has always been consistently fallacious. Two circles are called tangent when they have only one point in common. It follows then as a theorem that the line of centers passes through the point of contact. Wentworth has this theorem, page 91, but proves it by assuming it in his definition, page 75, §221. "Two circles are tangent to each other, *if both are tangent to a straight line* at the same point," which is of course only another form of the corollary: a perpendicular to the center-straight through the point of contact is a common tangent to the two circles.

The rest of the book is equally vulnerable, but, not thrice to slay the slain, we desist.

University of Texas.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

152. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

An operator on 'Change gains 5% on his *daily capital* every *odd* day of a business-week, and loses 5% of the same capital every *even* day of same week. What per cent. of his *original capital* will he have gained, or lost, at the end of a business-week?

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; HON. J. H. DRUMMOND, LL. D., Portland, Me.; and J. SCHEFFER, A. M., Hagerstown, Md.

Let 100%=original daily capital.

$100\% \times 1.05 = 105\%$=capital at beginning of second day.

$105\% \times .95 = 99.75\%$=capital at beginning of third day, and so on during the week.

$\therefore$ Saturday night he would have $100\% \times (1.05)^3 \times (95)^3 = (99.75)^3 = 99.25187\%$.

$\therefore$ $100\% - 99.25187\% = \frac{3}{4}\%$ nearly, his loss.

ALGEBRA.

130. Proposed by J. MARCUS BOORMAN, Woodmere, N. Y.

Solve $x^5 - y^5 = 2101$......(1), $x - y = 1$......(2). Find general formula for (1), ...(2), when $x^n - y^n = a$, $x - y = b$; for $n_0 = 3$; $n_1 = 5$; $n_2 = 7$, etc.

I. Solution by G.B.M.ZERR, A.M.. Ph.D., Professor of Chemistry and Physics in The Temple College. Philadelphia, Pa.

$(y+1)^5-y^5=2201.$

$(y^2+y)^2+y^2+y=420.$

$\therefore\ y^2+y=20$ or $-21.$

$y=4$ or -5 or $-\frac{1}{2}(1\mp\sqrt{-83}).$

$x=5$ or -4 or $\frac{1}{2}(1\pm\sqrt{-83}).$

II. Solution by the PROPOSER.

$5=n,\ \triangle=1.$ $\therefore$ Table (I), below, $1^5+5xy.1^3+5x^2y^2\ (1)=2101$; so therefore $5[x^2y^2.1^1+xy.1^3]=2101-1^5.$

$\therefore\ \frac{1}{5}[2101-1^5]=420$; hence $x^2y^2+xy+\frac{1}{4}=420+\frac{1}{4}$......(3), as $\triangle=1$, here.

$\therefore\ xy=-0.5\pm20.5$, and $4xy=80$ or $-84.$

$$(2)......(x-y)^2=\underline{\ 1\ }\qquad\underline{\ 1\ }$$

$$\therefore\ (x+y)^2=81 \text{ or } -83.$$

$\frac{1}{2}$(the root of (4)+(2)= $x=\frac{1}{2}[1\pm9]$ or $0.5[1\pm i\sqrt{83}$......(5).

$\frac{1}{2}$(the root of (4)−(2)= $y=\frac{1}{2}[-1\pm9]$ or $0.5[-1\pm i\sqrt{83}$, the eight roots.

That is, $x=5$ to $y=4$; $x=-4$ to $y=-5$; etc., as at (5).

Power Difference Theorems. Given $\triangle=x-y=b$, and any one of $w=xy$, or x^n-y^n, or $x^m+y^m,=a$, to find all by *quadratic* to $n=5$; *cubic* to $n=9$; *quintic* to $n=11$, etc. n=odd, viz., 3...5...7...9...11, etc.; m=even integers.

$$\text{(I).}\quad x^n-y^n=\Delta^n+nw\Delta^{n-2}+[\tfrac{1}{2}(n-1)(n-2)-(n-2)^0]w^2\Delta^{n-4}+$$
$$n[\tfrac{1}{6}(n-1)(n-2)-(n-3)]w^3\Delta^{n-6}+n[\tfrac{1}{24}(n-6)^3-(n-6)]w^4\Delta^{n-8}+$$
$$n[\tfrac{1}{120}(n-6)\{(n-8)^3-(n-8)\}]w^5\Delta^{n-10}+\ ......$$
$$+\tfrac{1}{24}(n^3-n)w^{\frac{1}{2}(n-3)}\Delta^3+nw^{\frac{1}{2}(n-1)}\Delta.$$

$$\text{(II).}\quad x^m+y^m=\Delta^m+mw\Delta^{m-2}+[\tfrac{1}{2}(m-1)(m-2)-(m-2)^0]w^2\Delta^{m-4}+......$$
$$\text{(like } n\text{)}......+\tfrac{1}{4}(m^2)w^{\frac{1}{2}(m-2)}\Delta^2+2w^{\frac{1}{2}m}.$$

Also solved by *H. C. WHITAKER*.

131. Proposed by HARRY S. VANDIVER, Bala. Pa.

It is well known that, when we define the symbol $\sqrt[n]{a}$ after the manner of elementary text-books on algebra, certain *irrational equations* may be written down which have no real or imaginary roots. Required then, the condition, if any, between a, b, c, and d such that the equation, $ax+b+\sqrt{(cx^2+d)}=0$, shall have no root, real or imaginary.

I. Solution by G. B. M. ZERR, A. M.. Ph.D.. Professor of Chemistry and Physics. The Temple College. Philadelphia, Pa.

$(a^2-c)x^2+2abx=d-b^2$, or $(c-a^2)x^2-2abx=b^2-d.$